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**NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS**

**REPORT No. 623**

**A STUDY OF THE TORQUE EQUILIBRIUM  
OF AN AUTOGIRO ROTOR**

**By F. J. BAILEY, Jr.**



**1938**

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# AERONAUTIC SYMBOLS

## 1. FUNDAMENTAL AND DERIVED UNITS

Symbol	Metric		English	
	Unit	Abbrevia- tion	Unit	Abbrevia- tion
Length Time Force	$l$ $t$ $F$	meter second weight of 1 kilogram	foot (or mile) second (or hour) weight of 1 pound	ft. (or mi.) sec. (or hr.) lb.
Power Speed	$P$ $V$	horsepower (metric) kilometers per hour meters per second	horsepower miles per hour feet per second	hp. m.p.h. f.p.s.

## 2. GENERAL SYMBOLS

$W$ , Weight= $mg$	$\eta$ , Kinematic viscosity
$g$ , Standard acceleration of gravity = 0.90685 m/s <sup>2</sup> or 32.1740 ft./sec. <sup>2</sup>	$\rho$ , Density (mass per unit volume)
$m$ , Mass = $\frac{W}{g}$	$\sigma$ , Standard density of dry air, 0.12497 kg-m <sup>-3</sup> -s <sup>2</sup> at 15° C. and 760 mm; or 0.002378 lb.-ft. <sup>-3</sup> -s <sup>2</sup>
$I$ , Moment of inertia = $mk^2$ . (Indicate axis of radius of gyration $k$ by proper subscript.)	$\gamma$ , Specific weight of "standard" air, 1.2255 kg/m <sup>3</sup> or 0.07651 lb./cu. ft.
$\mu$ , Coefficient of viscosity	

## 3. AERODYNAMIC SYMBOLS

$S$ , Area	$\alpha$ , Angle of setting of wings (relative to thrust line)
$S_w$ , Area of wing	$\alpha_s$ , Angle of stabilizer setting (relative to thrust line)
$C_l$ , Lift	$M$ , Resultant moment
$b$ , Span	$\omega$ , Resultant angular velocity
$c$ , Chord	$\frac{V}{\nu}$ , Reynolds Number, where $l$ is a linear dimension (e.g., for a model airfoil 3 in. chord, 100 m.p.h. normal pressure at 15° C., the corresponding number is 234,000; or for a model of 19 cm. chord, 40 m.p.s., the corresponding number is 274,000)
$\frac{b^2}{S}$ , Aspect ratio	$C_p$ , Center-of-pressure coefficient (ratio of distance of c.p. from leading edge to chord length)
$V$ , True air speed	$\alpha$ , Angle of attack
$q$ , Dynamic pressure = $\frac{1}{2}\rho V^2$	$\epsilon$ , Angle of downwash
$L$ , Lift, absolute coefficient $C_L = \frac{L}{qS}$	$a_o$ , Angle of attack, infinite aspect ratio
$D$ , Drag, absolute coefficient $C_D = \frac{D}{qS}$	$a_i$ , Angle of attack, induced
$D_o$ , Profile drag, absolute coefficient $C_{D_o} = \frac{D_o}{qS}$	$a_a$ , Angle of attack, absolute (measured from zero-lift position)
$D_i$ , Induced drag, absolute coefficient $C_{D_i} = \frac{D_i}{qS}$	$\gamma$ , Flight-path angle
$D_p$ , Parasite drag, absolute coefficient $C_{D_p} = \frac{D_p}{qS}$	
$C$ , Cross-wind force, absolute coefficient $C_C = \frac{C}{qS}$	
$R$ , Resultant force	

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**Langley Memorial Aeronautical Laboratory**

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### SUMMARY

*Two improvements have been made in the method developed in N. A. C. A. Reports Nos. 487 and 591 for the estimation of the inflow velocity required to overcome a given decelerating torque in an autogiro rotor. At low tip-speed ratios, where the assumptions necessary for the analytical integrations of the earlier papers are valid, the expressions therein derived are greatly simplified by combining and eliminating terms with a view to minimizing the numerical computations required. At high tip-speed ratios, by means of charts based on graphical integrations, errors inherent in the assumptions associated with the analytical method are largely eliminated.*

*The suggested method of estimating the inflow velocity presupposes a knowledge of the decelerating torque acting on the rotor; all available full-scale experimental information on this subject is therefore included.*

### INTRODUCTION

The results reported in reference 1 showed the agreement between the calculated and the experimental values of the forces and moments on an autogiro rotor to be unsatisfactory at tip-speed ratios greater than 0.3. An attempt to isolate the sources of the disagreement was made by comparing values of the analytically integrated expressions for thrust and accelerating torque from reference 1 with values obtained by graphical integration of the elemental forces. This comparison indicated that the simplifications introduced in reference 1 to make possible the analytical integration of the basic expression for accelerating torque are not valid at high tip-speed ratios. Consequently, the relation therein given between accelerating torque and inflow velocity results in an erroneous estimation of the inflow velocity required to overcome a given decelerating torque at high tip-speed ratios. Because the value of the inflow velocity so estimated must be used in calculating the rotor forces and moments from the other expressions of reference 1, this error introduces corresponding errors into the predicted values for all the various items constituting the rotor performance.

In the present paper an attempt is made to establish a more satisfactory method of expressing the relation between the accelerating torque and the inflow velocity, so that the inflow velocity required to overcome a given

decelerating torque at high tip-speed ratios may be readily estimated with an accuracy comparable with that previously attainable only at low tip-speed ratios. At the same time, the original method of estimating the inflow velocity at low tip-speed ratios is simplified by regrouping the terms in the torque equation in a manner designed to eliminate most of the tedious numerical computations previously required.

The method given presupposes a knowledge of the decelerating torque arising from the drag of the blade elements. Consequently all available full-scale experimental information on this subject is included as a guide to the designer until such time as a satisfactory method is devised for predicting decelerating torque from blade airfoil characteristics.

### ANALYSIS

#### GENERAL

The values of the expressions for the forces and moments on an autogiro rotor, as derived in references 1 and 2, are all critically dependent on the mean value of the inflow velocity through the rotor disk. Any attempt to predict the performance of a particular rotor therefore requires, first, an accurate determination of the inflow velocity necessary to maintain steady autorotation of the rotor under the given conditions. If use is made of the physical requirement that the net torque on the rotor must be zero in steady autorotation, the estimation of the inflow velocity may be accomplished in two steps: first, the estimation of the decelerating torque on the rotor due to the profile drag of the blade elements; and, second, the estimation of the inflow velocity required to generate in the rotor an accelerating torque just equal to the estimated decelerating torque. The present paper being primarily concerned with the prediction of the inflow velocity when the decelerating torque is known, the second of these steps will first be considered.

Throughout the following analysis it will be assumed, as in reference 1, that the inflow velocity is uniform over the disk area and that the contribution of the radial-velocity components to the elemental forces is negligible. The notation used is identical with that of reference 2; for convenience, however, a list of symbols and their definitions has been included.

## SYMBOLS AND DEFINITIONS

- $\Omega$ , rotor angular velocity, radians per second.  
 $R$ , blade radius.  
 $V$ , forward speed.  
 $\alpha$ , rotor angle of attack, radians.  
 $\lambda\Omega R$ , speed of axial flow through rotor.  
 $\mu\Omega R$ , component of forward speed in plane of disk.  
 $b$ , number of blades.  
 $\psi$ , blade azimuth angle measured from down wind in direction of rotation, radians.  
 $c$ , blade chord.  
 $B$ , tip-loss factor.  
 $xR$ , radius of blade element.  
 $u_r\Omega R$ , velocity component at blade element perpendicular to blade span and parallel to rotor disk.  
 $u_p\Omega R$ , velocity component at blade element perpendicular both to blade span and to  $u_r\Omega R$ .  
 $\phi = \tan^{-1} \frac{u_p}{u_r}$ .  
 $a$ , slope of lift coefficient against angle of attack of blade airfoil section (radian measure).  
 $Q$ , rotor torque.  
 $Q_a$ , accelerating rotor torque.  
 $Q_d$ , decelerating rotor torque.  
 $C_Q = \frac{Q}{\rho\Omega^2\pi R^3}$ .  
 $T$ , rotor thrust.  
 $C_T = \frac{T}{\rho\Omega^2\pi R^4}$ .  
 $\sigma$ , solidity, ratio of total blade area to swept-disk area,  $bc/\pi R$ .  
 $\theta_0$ , blade pitch angle at hub, radians.  
 $\theta_1$ , difference between hub and tip pitch angles, radians.  
 $\theta_e$ , equivalent constant pitch angle, radians.  
 $a_n$ , coefficient of  $\cos n\psi$  in Fourier series expressing blade flapping angle, radians.  
 $b_n$ , coefficient of  $\sin n\psi$  in Fourier series expressing blade flapping angle, radians.  
 $\epsilon_n$ , coefficient of  $\cos n\psi$  in Fourier series expressing periodic blade twist angle, radians.  
 $\eta_n$ , coefficient of  $\sin n\psi$  in Fourier series expressing periodic blade twist angle, radians.  
 $I_1$ , mass moment of inertia of rotor blade about horizontal hinge.  
 $\gamma = \frac{cpaR^4}{I_1}$ , mass constant of rotor blade.  
 $M_w$ , weight moment of blade about horizontal hinge.

## ACCELERATING TORQUE

The basic expression for the accelerating torque on an autogiro rotor is

$$Q_a = \frac{b}{2\pi} \frac{1}{\rho\Omega^2 R^4} \sum_0^{2\pi} \Delta\psi \sum_0^R u_r^2 C_L \frac{\sin \phi}{\cos^2 \phi} x \Delta x \quad (1)$$

By definition

$$Q_a = C_{Q_a} \rho \Omega^2 \pi R^5 \quad (4)$$

and

$$\sigma = \frac{bc}{\pi R}$$

Then

$$\frac{C_{Q_a}}{\sigma} = \frac{1}{2} \left( \frac{1}{2\pi} \sum_0^{2\pi} \Delta\psi \sum_0^R u_r^2 C_L \frac{\sin \phi}{\cos^2 \phi} x \Delta x \right) \quad (2)$$

The indicated summation may be performed analytically on the assumptions that  $C_L$  varies linearly with angle of attack of the element, regardless of the magnitude of this angle, and that the angle  $\phi$  is always so small that the sine, tangent, and angle are interchangeable. This integration has been covered in reference 1 for blades having a constant twist and has been extended in reference 2 to cover the more general case in which the blade twist varies periodically with the azimuth position of the blade. The final expression is:

$$\begin{aligned} \frac{C_{Q_a}}{\sigma} = & \frac{a}{2} \left[ \lambda^2 \left( \frac{1}{2} B^2 - \frac{1}{4} \mu^2 \right) + \lambda \left( \frac{1}{3} \theta_0 B^3 + \frac{2}{9} \mu^3 \theta_0 + \frac{1}{4} \theta_1 B^4 \right. \right. \\ & + \frac{1}{32} \mu^4 \theta_1 \left. \right) + \mu \lambda a_1 \left( \frac{1}{2} B^2 - \frac{3}{8} \mu^2 \right) + a_0^2 \left( \frac{1}{4} \mu^2 B^2 - \frac{1}{16} \mu^4 \right) \\ & - \frac{1}{3} \mu a_0 b_1 B^3 + a_1^2 \left( \frac{1}{8} B^4 + \frac{3}{16} \mu^2 B^2 \right) + b_1^2 \left( \frac{1}{8} B^4 + \frac{1}{16} \mu^2 B^2 \right) \\ & - a_2 \left( \frac{1}{4} \mu^2 a_0 B^2 + \frac{1}{6} \mu b_1 B^3 \right) + \frac{1}{2} a_2^2 B^4 \\ & + b_2 \left( \frac{1}{8} \mu^2 \theta_0 B^2 + \frac{1}{12} \mu^2 \theta_1 B^3 + \frac{1}{6} \mu a_1 B^3 \right) + \frac{1}{2} b_2^2 B^4 \\ & + \epsilon_0 \left( \frac{1}{4} \lambda B^4 + \frac{1}{12} \mu^2 b_2 B^3 + \frac{1}{32} \mu^4 \lambda \right) \\ & + \frac{1}{2} \epsilon_1 \left( -\frac{1}{4} \mu a_0 B^4 + b_1 \left[ \frac{1}{5} B^5 + \frac{1}{12} \mu^2 B^3 \right] - \frac{1}{8} \mu a_2 B^4 \right) \\ & + \frac{1}{2} \eta_1 \left( \frac{1}{3} \lambda \mu B^3 - a_1 \left[ \frac{1}{5} B^5 - \frac{1}{12} \mu^2 B^3 \right] - \frac{1}{8} \mu b_2 B^4 \right) \\ & + \frac{1}{2} \epsilon_1 \left( \frac{1}{4} \mu a_1 B^4 + \frac{2}{5} b_2 B^5 \right) \\ & + \frac{1}{2} \eta_2 \left( -\frac{1}{6} \mu^2 a_0 B^3 + \frac{1}{4} \mu b_1 B^4 - \frac{2}{5} a_2 B^5 \right) \left. \right\} \quad (3) \end{aligned}$$

When this expression is compared with equation (35) of reference 2, it must be remembered that the term  $\frac{\delta}{4a} \left( 1 + \mu^2 - \frac{1}{8} \mu^4 \right)$  in equation (35) of reference 2 is in reality  $2C_{Q_d}/\sigma a$ . This quantity will equal  $2C_{Q_d}/\sigma a$  in steady autorotation. Consequently,  $C_{Q_d}/\sigma$  will be  $a/2$  times the sum of the remaining terms in equation (35) of reference 2.

Expressions for the Fourier coefficients of blade flapping, also derived in reference 2 (equations (22), (23), (24), (27), and (28)), are:

$$\begin{aligned} a_0 = & \frac{1}{2} \gamma \left[ \frac{1}{3} \lambda B^3 + 0.080 \mu^3 \lambda + \frac{1}{4} \theta_0 \left( B^4 + \mu^2 B^2 - \frac{1}{8} \mu^4 \right) \right. \\ & + \frac{1}{5} (\theta_1 + \epsilon_0) \left( B^5 + \frac{5}{6} \mu^2 B^3 \right) + \frac{1}{8} \mu^2 b_2 B^3 + \frac{1}{4} \mu \eta_1 B^4 \\ & \left. - \frac{1}{12} \mu^2 \epsilon_0 B^3 \right] - \frac{M_w}{I_1 \Omega^2} \end{aligned} \quad (4)$$

$$a_1 = \frac{2\mu}{B^4 - \frac{1}{2}\mu^2 B^2} \left[ \lambda \left( B^2 - \frac{1}{4}\mu^2 \right) + \theta_0 \left( \frac{4}{3}B^3 + 0.106\mu^3 \right) + (\theta_1 + \epsilon_0)B^4 - \frac{1}{3}b_2 B^3 + \frac{\eta_1}{\mu} \left( \frac{2}{5}B^5 + \frac{1}{2}\mu^2 B^3 \right) - \frac{1}{2}\epsilon_2 B^4 \right] \quad (5)$$

$$b_1 = \frac{4\mu}{B^4 + \frac{1}{2}\mu^2 B^2} \left[ a_0 \left( \frac{1}{3}B^3 + 0.035\mu^3 \right) + \frac{1}{6}a_2 B^3 - \frac{\epsilon_1}{\mu} \left( \frac{1}{5}B^5 + \frac{1}{12}\mu^2 B^3 \right) - \frac{1}{4}\eta_2 B^4 \right] \quad (6)$$

$$a_2 = \frac{\mu^2 \gamma}{\gamma^2 B^8 + 144} \left\{ \lambda B \left( 16 + \frac{7}{108} \gamma^2 B^8 \right) + \theta_0 B^2 \left( \frac{46}{3} + \frac{7}{144} \gamma^2 B^8 \right) + (\theta_1 + \epsilon_0) B^3 \left( 12 + \frac{7}{180} \gamma^2 B^8 \right) - \frac{1}{30} \frac{\epsilon_1}{\mu} \gamma B^8 + \frac{2}{5} \frac{\eta_1}{\mu} B^4 + \frac{24}{5} \frac{\epsilon_2}{\mu^2} B^5 + \frac{2}{5} \frac{\eta_2}{\mu^2} \gamma B^6 \right\} \quad (7)$$

$$b_2 = \frac{-\mu^2 \gamma}{\gamma^2 B^8 + 144} \left\{ \frac{5}{9} \lambda B^5 + \frac{25}{36} \theta_0 B^6 + \frac{8}{15} (\theta_1 + \epsilon_0) B^7 + \frac{2}{5} \frac{\epsilon_1}{\mu} B^4 + \frac{1}{30} \frac{\eta_1}{\mu} B^8 + \frac{2}{5} \frac{\epsilon_2}{\mu^2} B^9 - \frac{24}{5} \frac{\eta_2}{\mu^2} \gamma B^6 \right\} \quad (8)$$

Substituting from equations (4), (5), (6), (7), and (8) into equation (3), retaining only terms of the order of  $\mu^4$  or lower, and regrouping

$$\begin{aligned} \frac{C_{a_2}}{\sigma} = & \frac{a}{2} \left[ K_1 \lambda^2 + \left[ K_2 \theta_0 + K_3 (\theta_1 + \epsilon_0) + K_7 \eta_1 + K_{10} \epsilon_1 + K_{11} \epsilon_2 \right. \right. \\ & + K_{12} \eta_2 + K_{24} \frac{M_w}{I_1 \Omega^2} \left. \right] \lambda + K_4 \theta_0^2 + K_5 \theta_0 (\theta_1 + \epsilon_0) \\ & + K_6 (\theta_1 + \epsilon_0)^2 + K_8 \theta_0 \eta_1 + K_9 (\theta_1 + \epsilon_0) \eta_1 + K_{13} \theta_0 \epsilon_1 \\ & + K_{14} \theta_0 \epsilon_2 + K_{15} \theta_0 \eta_2 + K_{16} (\theta_1 + \epsilon_0) \epsilon_1 + K_{17} (\theta_1 + \epsilon_0) \epsilon_2 \\ & + K_{18} (\theta_1 + \epsilon_0) \eta_2 + K_{19} \epsilon_1^2 + K_{20} \epsilon_1 \eta_1 + K_{21} \eta_1^2 \\ & + K_{22} (\epsilon_1 \epsilon_2 + \eta_1 \eta_2) + K_{23} (\epsilon_1 \eta_2 - \eta_1 \epsilon_2) + \frac{M_w}{I_1 \Omega^2} \left[ K_{25} \theta_0 \right. \\ & + K_{26} (\theta_1 + \epsilon_0) + K_{27} \epsilon_1 + K_{28} \eta_1 + K_{29} \epsilon_2 + K_{30} \eta_2 \\ & \left. \left. + K_{31} \frac{M_w}{I_1 \Omega^2} \right] \right] \quad (9) \end{aligned}$$

where the coefficients  $K_1$  to  $K_{31}$ , for which complete expressions are given in the appendix, are functions of  $\mu$ ,  $B$ , and  $\gamma$  only.

The magnitudes of the coefficients  $K_{24}$  to  $K_{31}$  are such that, when a value less than 0.01 is assigned to  $M_w/I_1 \Omega^2$ , the resulting contribution of the terms involving these coefficients to  $C_{a_2}/\sigma$  is negligible over the entire range of tip-speed ratios. The influence of  $\gamma$  on  $C_{a_2}/\sigma$  is also of a secondary nature; comparison of values of the coefficients  $K_1$  to  $K_{23}$  for  $\gamma=0$  with those for  $\gamma=15$  indicates that no appreciable error will arise from the use of a value of  $\gamma=15$  for any conventional rotor at any attainable altitude. Inasmuch as the chord-span

ratio of the blades used in present-day rotors varies only within small limits, it is possible to assign arbitrarily a value of 0.970 to the tip-loss factor  $B$  with the assurance that only minor departures from the recommended value of  $1 - \frac{1}{2} \frac{c}{R}$  (reference 1) will result.

By the use of the foregoing values for  $\gamma$  and  $B$  with extreme values of tip-speed ratio, pitch, twist, and inflow velocity, it can be shown that the terms involving  $K_{10}$ ,  $K_{13}$ ,  $K_{16}$ ,  $K_{19}$ ,  $K_{20}$ ,  $K_{22}$ , and  $K_{23}$  are negligible. Of the remaining coefficients,  $K_{11}$ ,  $K_{12}$ ,  $K_{14}$ ,  $K_{15}$ ,  $K_{17}$ ,  $K_{18}$ , and  $K_{21}$  are merely constants multiplied by  $\mu^2$ ; whereas,  $K_1$  to  $K_9$  involve two or more powers of  $\mu$ .

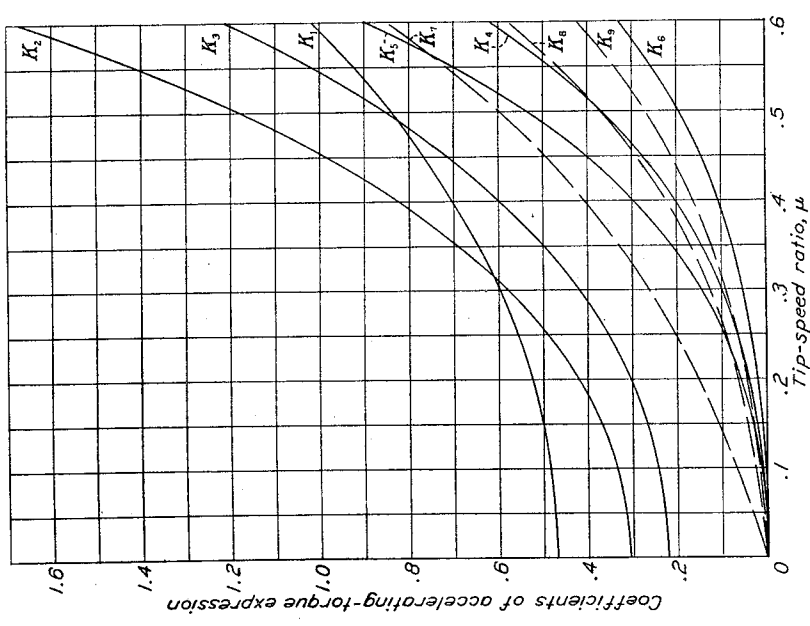
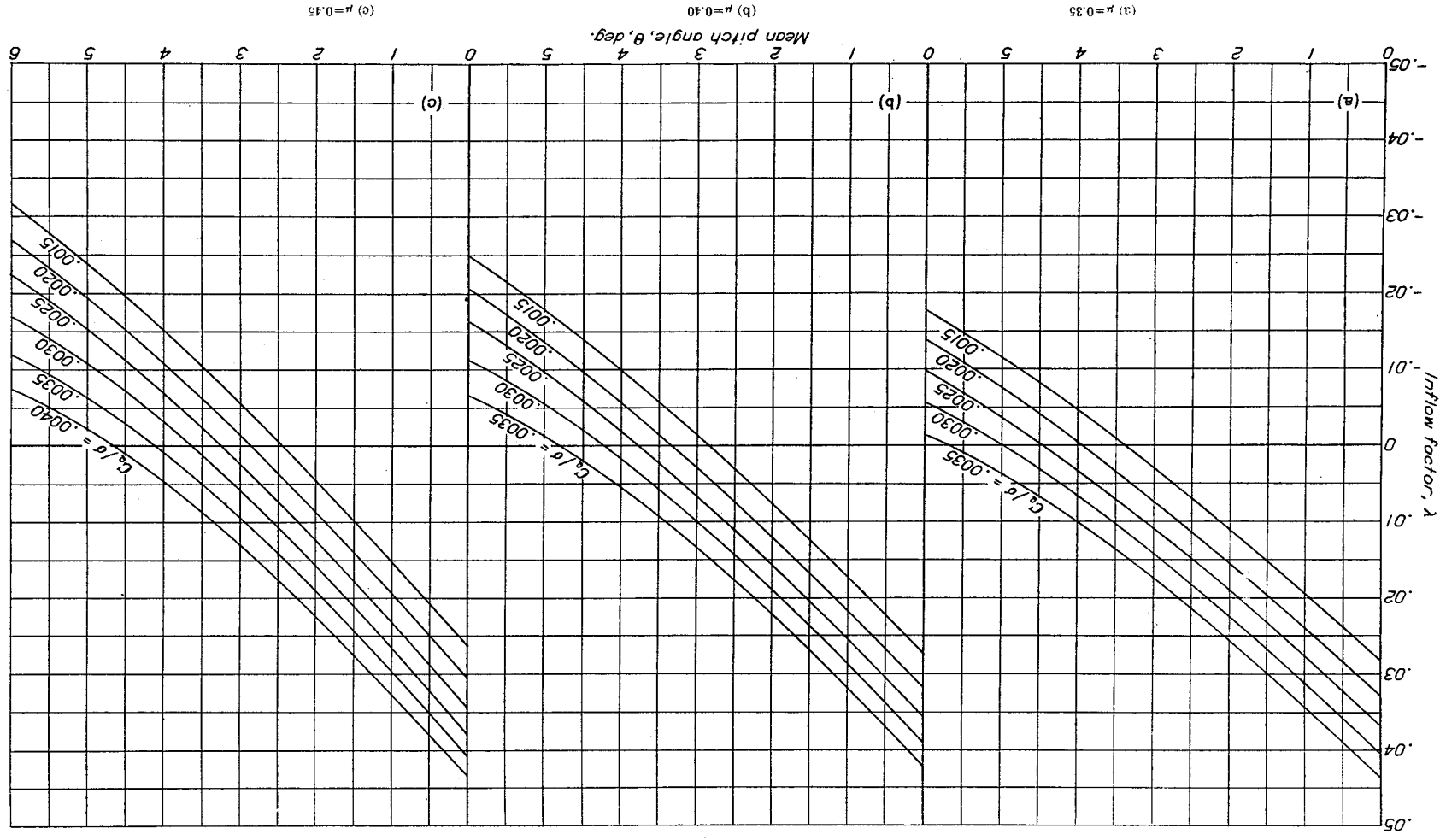


FIGURE 1.—Coefficients  $K_1$  to  $K_9$  of accelerating-torque expression.

Expression (9) may then be reduced to

$$\begin{aligned} \frac{C_{a_2}}{\sigma} = & \frac{a}{2} \left\{ K_1 \lambda^2 + [K_2 \theta_0 + K_3 (\theta_1 + \epsilon_0) + K_7 \eta_1 \right. \\ & - (0.614\epsilon_2 + 0.141\eta_2) \mu^2] \lambda + K_4 \theta_0^2 \\ & + K_5 \theta_0 (\theta_1 + \epsilon_0) + K_6 (\theta_1 + \epsilon_0)^2 + K_8 \theta_0 \eta_1 \\ & + K_9 (\theta_1 + \epsilon_0) \eta_1 - [(0.284\epsilon_2 + 0.059\eta_2) \theta_0 \\ & \left. + (0.205\epsilon_2 + 0.047\eta_2) (\theta_1 + \epsilon_0) - 0.260\eta_1^2] \mu^2 \right\} \quad (10) \end{aligned}$$

Values of coefficients  $K_1$  to  $K_9$  may be found directly from figure 1 or for specified tip-speed ratios from table I.





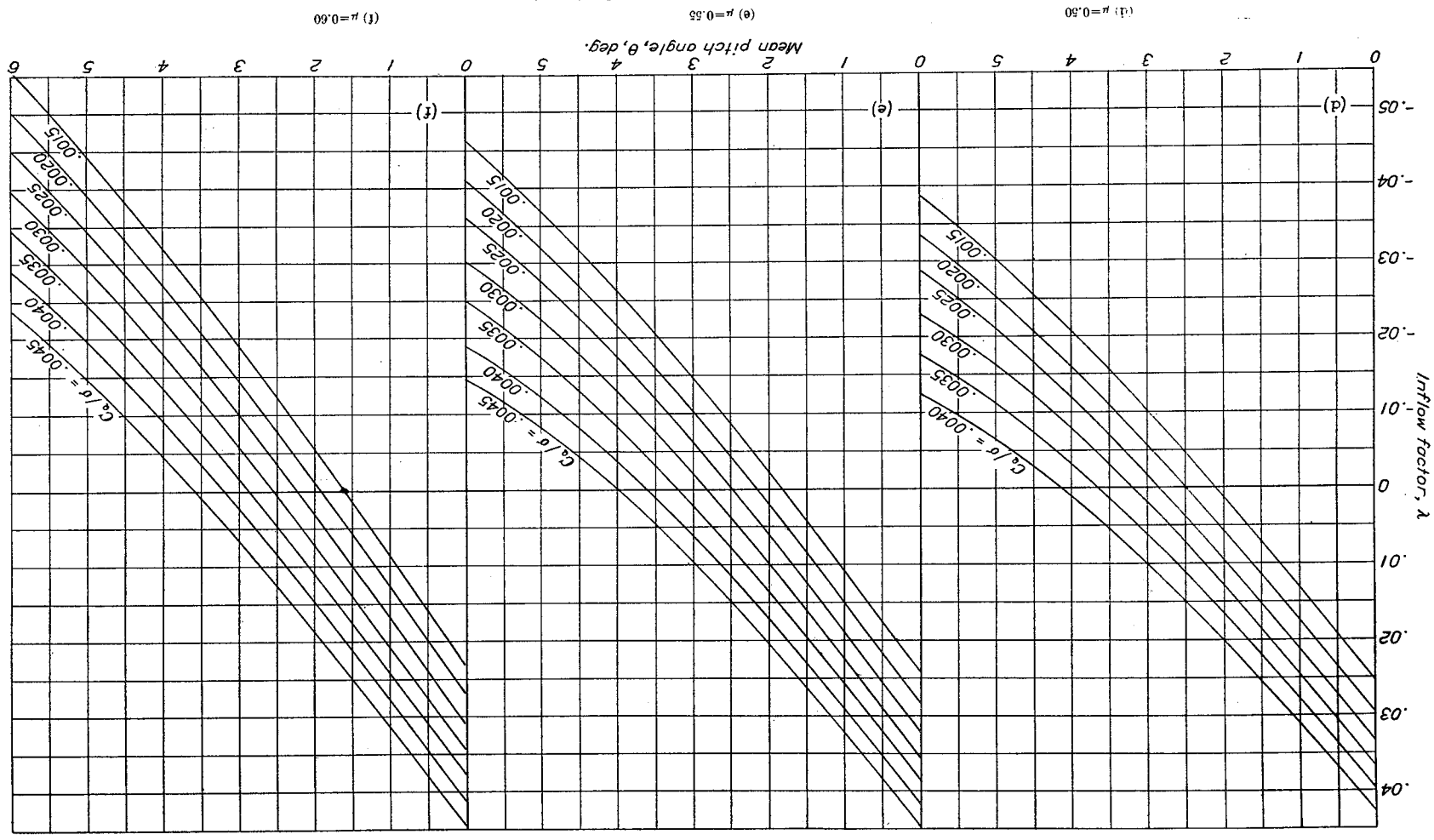


FIGURE 2.—Charts for determination of the inflow factor,  $\lambda$ .

Substitution of numerical values for the pitch setting and twist coefficients in equation (10) results in a quadratic in  $\lambda$  that may be solved for  $\lambda$  at any given value of  $C_{qa}/\sigma$ .

For rotors in which the pitch is constant along the radius of the blade and does not vary with azimuth position of the blade, equation (10) reduces to the form

$$\frac{C_{qa}}{\sigma} = \frac{a}{2} \{ K_1 \lambda^2 + K_2 \theta \lambda + K_3 \theta^2 \} \quad (11)$$

At tip-speed ratios above 0.4 with low pitch settings or above 0.3 with very high pitch settings, equations (9), (10), and (11) are inaccurate. The omission of terms of higher order than  $\mu^4$  becomes questionable and the overestimation of the lift coefficients of the stalled elements introduces a very serious error. Errors arising from the assumption that the angle  $\phi$  is small are also too large to ignore. Consequently, some other method of integrating the basic expression must be found to extend the relation between torque and inflow to higher tip-speed ratios.

Reference to equation (2) shows that the indicated summation may also be made by evaluating the torque for a series of elements throughout the disk and integrating graphically over the disk area. This summation is accomplished for any combination of tip-speed ratio, pitch setting, and inflow velocity by first calculating  $u_P$  and  $u_T$ , the perpendicular and tangential components, respectively, of the velocity relative to each chosen element. The angle  $\phi$  of the resultant velocity at each element to the plane of the disk is then  $\tan^{-1} u_P/u_T$ . Adding to  $\phi$  the angle  $\theta$  of the chord of the element to the plane of the disk, as defined by  $\theta_0, \theta_1, \epsilon_0, \epsilon_1, \eta_1, \epsilon_2$ , and  $\eta_2$ , gives the angle of attack of each element. From a lift curve of the airfoil section, the corresponding lift coefficient of the element is determined. Where  $u_T$  is negative, the use of  $-\phi - \theta$  in place of  $\phi + \theta$  allows for the effect of the reversed flow over the blade in the same manner in which this problem was handled in the analytical expressions of references 1 and 2.

For rotors without blade twist, a sufficient number of graphical integrations of this type have been carried out to permit the construction of the charts of figure 2 from which the value of  $\lambda$  corresponding to a given  $C_{qa}/\sigma$  may be directly determined. Because accelerating and decelerating torques are equal in steady autorotation, the subscript has been omitted from the torque coefficient in figure 2. Comparison of the values of  $\lambda$  given by the charts with those obtained from the analytical expression (11) at the same  $C_{qa}/\sigma$  will demonstrate the error in expression (11). As in the analytical expression, values of  $\gamma = 1.5$  and  $B = 0.97$  have been used throughout the graphical integrations in the belief that the generality of the results is not appreciably impaired by these substitutions. A further limitation of the generality of the charts, imposed by the use throughout the graphical integrations of the lift characteristics of the N. A. C. A. 0012 airfoil section, is not considered

serious because only small departures from these characteristics are to be expected in blade sections commonly used for rotors at the present time.

Obviously, it is impossible to supply charts covering all possible combinations of fixed and varying twist. It is reasonable to believe, however, that equations (10) and (11), although quantitatively in error as to the total accelerating torque produced by a given inflow velocity, correctly express the relative merit of rotors with and without twist as torque-generating devices. This belief amounts merely to the assumption that the percentage error inherent in the analytical method is the same for rotors with twist as for those without twist. Hence, it should be satisfactory at high tip-speed ratios to use equations (10) and (11) solely as a means of determining, for any rotor with twist, the pitch of what may be termed an "equivalent" constant-pitch rotor, that is, a rotor with no twist capable of generating the same accelerating torque at the same inflow velocity. The charts (fig. 2) for constant-pitch rotors may then be used to find the true value of the inflow velocity for the equivalent constant-pitch rotor, and the value so found may be considered to apply also to the original twisted rotor. In detail the procedure is as follows:

1. Substitute values for  $C_{qa}/\sigma, a, \mu, K_1$  to  $K_5, \theta_0, \theta_1, \epsilon_0, \eta_1, \epsilon_2$ , and solve equation (10) for  $\lambda$ .
2. Using this value of  $\lambda$  in equation (11), together with the original value of  $C_{qa}/\sigma$ , solve equation (11) for  $\theta$ . The value of  $\theta$  obtained is then the pitch of a constant-pitch rotor capable of generating the required torque at the same inflow velocity as the original rotor.
3. From the charts for constant-pitch rotors (fig. 2), determine the true inflow factor  $\lambda$  for a constant-pitch rotor of pitch  $\theta$  generating torque  $C_{qa}/\sigma$ . The value of  $\lambda$  for the original rotor is, by hypothesis, identical with that of the equivalent constant-pitch rotor as found in step 3.

#### DECELERATING TORQUE

It will be noted that, in order to estimate the inflow velocity by the method of the preceding section, it is necessary to know the value of the accelerating torque  $C_{qa}/\sigma$ . On the strength of the physical requirement that accelerating and decelerating torques must be equal in steady autorotation, this quantity is normally determined by estimating the decelerating torque on the rotor from an integration over the disk area of the decelerating torque arising from the profile drag of a blade element. Hence

$$-\frac{C_{da}}{\sigma} = -\frac{C'_{da}}{\sigma} = -\frac{1}{2} \left( \frac{1}{2\pi} \int_0^{2\pi} d\psi \int_0^r u_r^2 C_n \sec \phi dx \right) \quad (12)$$

The rigorous evaluation of the integral of expression (12) is possible only by graphical means because the elemental profile-drag coefficient  $C_n$  is a nonlinear function of both the angle of attack and the Reynolds Number at the element. Even then, uncertainty concerning the value of  $C_D$  in the presence of high radial velocities or in the stalled portion of the disk makes the validity

of the results extremely doubtful. Nor does there appear to be any merit in the idea of replacing  $C_D \sec \phi$  by a mean value  $\delta$ , as was done in reference 1, so that

$$\frac{1}{2\pi} \int_0^{2\pi} d\psi \int_0^1 u_T^2 C_D \sec \phi dx = \frac{1}{2\pi} \int_0^{2\pi} d\psi \int_0^1 u_T^2 dx \\ = \delta \left( \frac{1}{4} + \frac{1}{4} \mu^2 \right)$$

or, when the effect of the reversed-velocity region is included as in reference 1,

$$= \delta \left( \frac{1}{4} + \frac{1}{4} \mu^2 - \frac{1}{32} \mu^4 \right) \quad (13)$$

because both graphical integration and experiment indicate that the magnitude of  $\delta$  will vary with both pitch

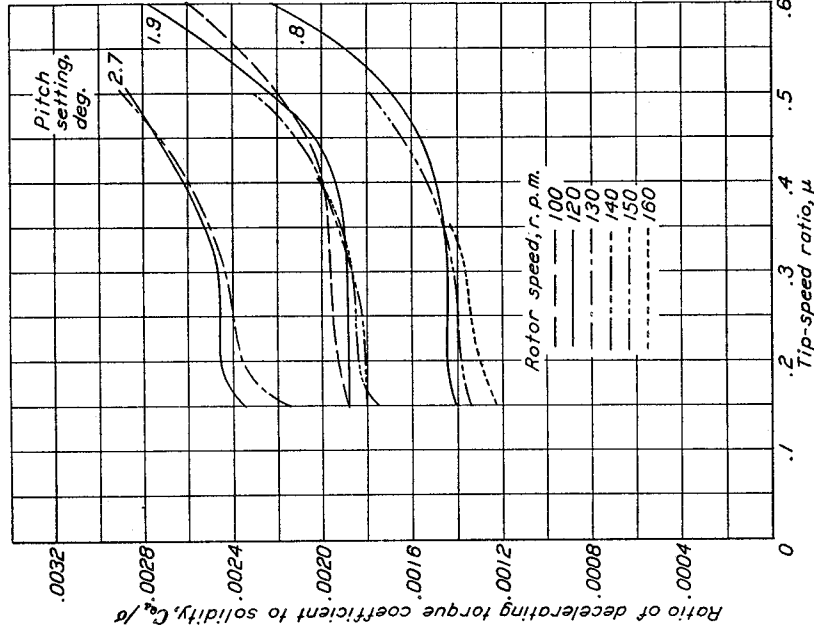


FIGURE 3.—Experimental decelerating torque of the PCA-2 rotor.

setting and tip-speed ratio and hence will require empirical information for its estimation. It seems fully as desirable and much simpler to formulate the empirical rules for the direct estimation of  $C_{qd}/\sigma$ . Unfortunately, the only reliable full-scale experimental information available for this purpose is confined to a single rotor, the PCA-2; and it is obviously impossible to establish such rules with any degree of finality at the present time. It does seem desirable, however, to study these data rather thoroughly in an effort to develop a tentative method of estimating  $C_{qd}/\sigma$  that may be expected to be subject to only minor alterations as additional data are accumulated.

Experimental data on  $C_{qd}/\sigma$ , from full-scale wind-tunnel tests of the PCA-2 rotor, are shown in figure 3. The values given were obtained by substituting experimental values of thrust and blade-motion coefficients in the thrust-coefficient expression of reference 2 and solving for  $\lambda$ . The value of  $C_{qd}/\sigma$ , and hence of  $C_{qd}/\sigma$ , corresponding to this value of  $\lambda$  was then obtained by a process the reverse of the one described in the preceding section of this paper.

A comparison of the values of  $C_{qd}/\sigma$  given in figure 3 with the experimental values of the ratio  $C_T/\sigma$  shown in figure 4 reveals that, at any given tip-speed ratio,  $C_{qd}/\sigma$  at all the pitch settings tested is almost directly proportional to  $C_T/\sigma$ . The closeness of this relationship is indicated in figure 5 where experimental values of the

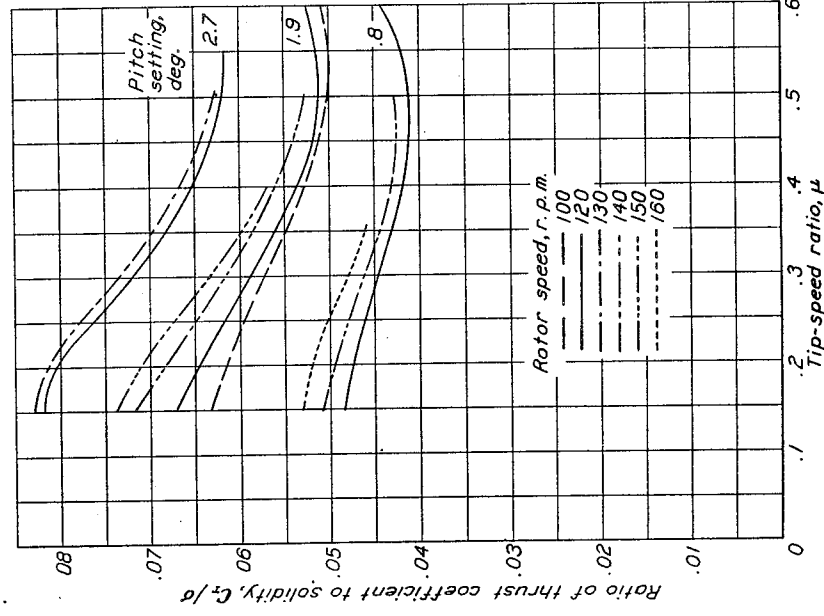


FIGURE 4.—Experimental thrust coefficient of the PCA-2 rotor.

quotient  $C_q/C_T$  are plotted as functions of  $\mu$  for the pitch settings given in figures 3 and 4.

From the designer's viewpoint, the curve shown in figure 5 provides a possible, although tedious, method of estimating the performance of a new design. As the various steps involved may not be instantly apparent, it seems desirable to outline briefly the recommended procedure.

Normally, from considerations of rotor efficiency, it is mandatory that the designer secure some definite value of  $C_T/\sigma$  at a particular tip-speed ratio. From this point, the problem becomes one of determining, first,

the pitch angle required to satisfy this condition; and second, the performance of a rotor of this pitch as a function of the tip-speed ratio. On the assumption that the geometric and aerodynamic characteristics of the blades are known, the steps required in the solution are as follows:

1. Compute  $(\theta_1 + \epsilon_0)$ ,  $\eta_1$ ,  $\epsilon_2$ , and  $\eta_2$  from the expressions of reference 3, neglecting the influence of the  $\lambda$ ,  $\theta_0$ , and  $(\theta_1 + \epsilon_0)$  terms.
2. From  $C_T/\sigma$  and  $\mu$  find  $C_{Q_d}/\sigma$  from figure 5.
3. Assume a series of values of  $\theta_0$  and, using  $C_{Q_d}/\sigma$  from step 2, find the corresponding values of  $\lambda$  by the method described in the preceding section of this paper.
4. By substituting values of  $\lambda$  and  $\theta_0$  from step 3 into the thrust expression of reference 2, ignoring the terms

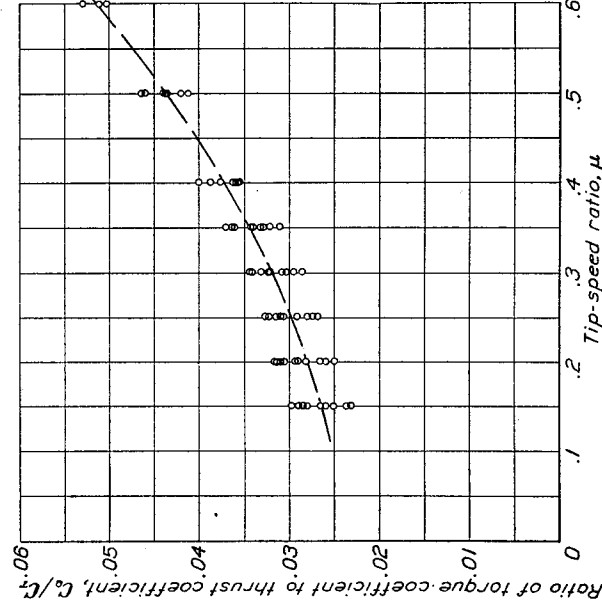


FIGURE 5.—Experimental torque-thrust ratio,  $C_Q/C_T$ , for the PCA-2 rotor.

involving  $a_1$  and  $b_2$ , which almost cancel one another, obtain  $C_T/\sigma$  as a function of  $\theta_0$ .

5. Fix  $\theta_0$  at the value corresponding to the required  $C_T/\sigma$ .

The twist coefficients may now be recalculated without neglecting the  $\lambda$ ,  $\theta_0$ , and  $\theta_1$  terms and the entire process repeated to determine more accurately the required  $\theta_0$ . The change in  $\theta_0$  resulting from this second approximation may be expected to be very small since the twist coefficients are only slightly affected by inflow velocity and pitch. The small change can probably be safely neglected except in unusual designs.

It will be noted, of course, that at low tip-speed ratios, where equations (10) and (11) are valid,  $\lambda$  may be obtained in step 3 as an analytical function of  $\theta_0$ ; this function may then be substituted into the expression for  $C_T/\sigma$  in step 4. The result will be a quadratic in  $\theta_0$  that can be solved directly for  $\theta_0$  at the design value of  $C_T/\sigma$ . Although this approach appears, at first glance, to be the more direct, in actual practice it will be found simpler to proceed as originally outlined, even at low tip-speed ratios.

The performance of the rotor of pitch  $\theta_0$  at other tip-speed ratios can be determined as follows:

1. Assume a value of  $C_T/\sigma$  at the new tip-speed ratio. Figure 4 may be used as a guide to a reasonable choice.
2. Obtain the corresponding  $C_{Q_d}/\sigma$  from figure 5.
3. Calculate  $\epsilon_0$ ,  $\eta_1$ ,  $\epsilon_2$ , and  $\eta_2$  for the new tip-speed ratio by the method of reference 3, neglecting terms involving  $\lambda$ .
4. Determine  $\lambda$  by the method described in the first part of this paper.
5. Calculate  $C_T/\sigma$  from the thrust-coefficient expression given in reference 2 and check against the value assumed in step 1.
6. Repeat, modifying the assumed value until agreement is obtained.

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,  
LANGLEY FIELD, VA., January 10, 1938.

# APPENDIX

Complete expressions follow for the coefficients  $K_1$  to  $K_{31}$  appearing in equation (9). For convenience of reference, the term to which each coefficient is attached is given in parentheses immediately preceding the expression for the coefficient.

$$\begin{aligned}
 (\lambda^2), K_1 &= \frac{1}{2}B^2 + \left(\frac{5}{4} + \frac{1}{1296}\gamma^2B^8\right)\mu^2 + \left[\frac{1}{2B^2} + \frac{\gamma^2B^6}{(144 + \gamma^2B^8)^2} \left(\frac{4}{3} - \frac{37}{162}\gamma^2B^8 - \frac{77}{46656}\gamma^4B^{16}\right)\right]\mu^4 \\
 (\lambda\theta_0), K_2 &= \frac{1}{3}B^3 + \left(\frac{8}{3}B + \frac{1}{864}\gamma^2B^9\right)\mu^2 + \frac{2}{9}\mu^3 + \left[\frac{8}{3B} + \frac{\gamma^2B^7}{(144 + \gamma^2B^8)^2} \left(\frac{224}{9} - \frac{11}{648}\gamma^2B^8 - \frac{41}{31104}\gamma^4B^{16}\right)\right]\mu^4 \\
 (\lambda[\theta_1 + \epsilon_0]), K_3 &= \frac{1}{4}B^4 + \left(2B^2 + \frac{1}{1080}\gamma^2B^{10}\right)\mu^2 + \left[\frac{65}{32} + \frac{\gamma^2B^8}{(144 + \gamma^2B^8)^2} \left(\frac{236}{15} - \frac{7}{108}\gamma^2B^8 - \frac{47}{38880}\gamma^4B^{16}\right)\right]\mu^4 \\
 (\theta_0^2), K_4 &= \left(\frac{8}{9}B^2 + \frac{1}{2304}\gamma^2B^{10}\right)\mu^2 + \left[\frac{20}{9} + \frac{\gamma^2B^8}{(144 + \gamma^2B^8)^2} \left(\frac{305}{36} + \frac{65}{1296}\gamma^2B^8 - \frac{5}{82944}\gamma^4B^{16}\right)\right]\mu^4 \\
 (\theta_0[\theta_1 + \epsilon_0]), K_5 &= \left(\frac{4}{3}B^3 + \frac{1}{1440}\gamma^2B^{11}\right)\mu^2 + \left[\frac{10}{3}B + \frac{\gamma^2B^9}{(144 + \gamma^2B^8)^2} \left(\frac{57}{5} + \frac{7}{144}\gamma^2B^8 - \frac{11}{51840}\gamma^4B^{16}\right)\right]\mu^4 \\
 ([\theta_1 + \epsilon_0]^2), K_6 &= \left(\frac{1}{2}B^4 + \frac{1}{3600}\gamma^2B^{12}\right)\mu^2 + \left[\frac{5}{4}B^2 + \frac{\gamma^2B^{10}}{(144 + \gamma^2B^8)^2} \left(\frac{92}{25} + \frac{1}{150}\gamma^2B^8 - \frac{17}{129600}\gamma^4B^{16}\right)\right]\mu^4 \\
 (\lambda\eta_1), K_7 &= \frac{23}{30}B^3\mu + \left[\frac{11}{6}B + \frac{1}{864}\gamma^2B^9 - \frac{1}{144 + \gamma^2B^8} \frac{49}{2160}\gamma^2B^9 + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{32}{5}\gamma^2B^9 + \frac{2}{45}\gamma^4B^{17}\right)\right]\mu^3 \\
 (\theta_0\eta_1), K_8 &= \frac{4}{15}B^4\mu + \left[\frac{89}{45}B^2 + \frac{1}{1152}\gamma^2B^{10} - \frac{1}{144 + \gamma^2B^8} \frac{53}{1728}\gamma^2B^{10} + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{92}{15}\gamma^2B^{10} + \frac{23}{540}\gamma^4B^{18}\right)\right]\mu^3
 \end{aligned}$$

$$\begin{aligned}
 ([\theta_1 + \epsilon_0]\eta_1), K_9 &= \frac{1}{5}B^5\mu + \left[\frac{89}{60}B^3 + \frac{1}{1440}\gamma^2B^{11} - \frac{1}{144 + \gamma^2B^8} \frac{43}{1800}\gamma^2B^{11} + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{24}{5}\gamma^2B^{11} + \frac{1}{30}\gamma^4B^{19}\right)\right]\mu^3 \\
 (\lambda\epsilon_1), K_{10} &= \frac{1}{720}\gamma^2B^7\mu + \left[-\frac{1}{540}\gamma^2B^5 + \frac{1}{144 + \gamma^2B^8} \left(\frac{1}{5}\gamma^2B^5 + \frac{5}{1728}\gamma^3B^{13}\right) - \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{14}{45}\gamma^3B^{13} + \frac{7}{3240}\gamma^5B^{21}\right)\right]\mu^3 \\
 (\lambda\epsilon_2), K_{11} &= \left[-\frac{3}{4}B^2 - \frac{1}{144 + \gamma^2B^8} \frac{16}{45}\gamma^2B^{10} + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{384}{5}\gamma^2B^{10} + \frac{8}{15}\gamma^4B^{18}\right)\right]\mu^2 \\
 (\lambda\eta_2), K_{12} &= \left[\frac{1}{72}\gamma^2B^6 - \frac{1}{144 + \gamma^2B^8} \left(\frac{24}{5}\gamma^2B^6 + \frac{2}{45}\gamma^3B^{14}\right) + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{56}{15}\gamma^3B^{14} + \frac{7}{270}\gamma^5B^{22}\right)\right]\mu^2 \\
 (\theta_0\epsilon_1), K_{13} &= \frac{1}{960}\gamma^2B^6\mu + \left[-\frac{1}{2880}\gamma^2B^6 + \frac{1}{144 + \gamma^2B^8} \left(\frac{1}{72}\gamma^2B^6 + \frac{5}{2304}\gamma^3B^{14}\right) - \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{7}{30}\gamma^3B^{14} + \frac{7}{4320}\gamma^5B^{22}\right)\right]\mu^3 \\
 (\theta_0\epsilon_2), K_{14} &= \left[-\frac{1}{3}B^3 - \frac{1}{144 + \gamma^2B^8} \frac{17}{36}\gamma^2B^{11} + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{368}{5}\gamma^2B^{11} + \frac{23}{45}\gamma^4B^{19}\right)\right]\mu^2 \\
 (\theta_0\eta_2), K_{15} &= \left[\frac{1}{96}\gamma^2B^7 - \frac{1}{144 + \gamma^2B^8} \left(\frac{37}{15}\gamma^2B^7 + \frac{1}{30}\gamma^3B^{15}\right) + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{42}{15}\gamma^3B^{15} + \frac{7}{360}\gamma^5B^{23}\right)\right]\mu^2 \\
 ([\theta_1 + \epsilon_0]\epsilon_1), K_{16} &= \frac{1}{1200}\gamma^2B^9\mu + \left[-\frac{1}{2400}\gamma^2B^7 + \frac{1}{144 + \gamma^2B^8} \left(\frac{1}{60}\gamma^2B^7 + \frac{1}{576}\gamma^3B^{15}\right) - \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{14}{75}\gamma^3B^{15} + \frac{7}{5400}\gamma^5B^{23}\right)\right]\mu^3 \\
 ([\theta_1 + \epsilon_0]\epsilon_2), K_{17} &= \left[-\frac{1}{4}B^4 - \frac{1}{144 + \gamma^2B^8} \frac{11}{30}\gamma^2B^{12} + \frac{1}{(144 + \gamma^2B^8)^2} \left(\frac{288}{5}\gamma^2B^{12} + \frac{2}{5}\gamma^4B^{20}\right)\right]\mu^2
 \end{aligned}$$



$$(\theta_1 + \epsilon_0 \eta_2), K_{18} = \left[ \frac{1}{120} \gamma B^8 - \frac{1}{144 + \gamma^2 B^8} \left( 2\gamma B^8 + \frac{2}{75} \gamma^3 B^{16} \right) + \frac{1}{(144 + \gamma^2 B^8)^2} \left( \frac{56}{25} \gamma^3 B^{16} + \frac{7}{450} \gamma^5 B^{24} \right) \right] \mu^2$$

$$(\epsilon_1^2), K_{19} = \left[ -\frac{1}{144 + \gamma^2 B^8} \frac{1}{7200} \gamma^2 B^{12} + \frac{1}{(144 + \gamma^2 B^8)^2} \left( \frac{2}{25} \gamma^2 B^{12} + \frac{1}{1800} \gamma^4 B^{20} \right) \right] \mu^2$$

$$(\epsilon_1 \eta_1), K_{20} = \frac{1}{960} \gamma B^8 \mu^2$$

$$(\eta_1^2), K_{21} = \left[ \frac{22}{75} B^4 - \frac{1}{144 + \gamma^2 B^8} \frac{1}{7200} \gamma^2 B^{12} + \frac{1}{(144 + \gamma^2 B^8)^2} \left( \frac{2}{25} \gamma^2 B^{12} + \frac{1}{1800} \gamma^4 B^{20} \right) \right] \mu^2$$

$$(\epsilon_1 \epsilon_2 + \eta_1 \eta_2), K_{22} = -\frac{1}{144 + \gamma^2 B^8} \frac{3}{50} \gamma B^9 \mu$$

$$(\epsilon_1 \eta_2 - \eta_1 \epsilon_2), K_{23} = \left[ \frac{1}{144 + \gamma^2 B^8} \frac{1}{120} \gamma^2 B^{13} - \frac{1}{(144 + \gamma^2 B^8)^2} \left( \frac{48}{25} \gamma^2 B^{13} + \frac{1}{75} \gamma^4 B^{21} \right) \right] \mu$$

$$\left( \lambda \frac{M_w}{I_1 \Omega^2} \right), K_{24} = -\frac{1}{108} \gamma B^8 \mu^2$$

$$+ \frac{1}{144 + \gamma^2 B^8} \left( \frac{47}{9} \gamma B^3 + \frac{7}{486} \gamma^3 B^{11} \right) \mu^4$$

$$\left( \theta_0 \frac{M_w}{I_1 \Omega^2} \right), K_{25} = -\frac{1}{144} \gamma B^8 \mu^2$$

$$+ \frac{1}{144 + \gamma^2 B^8} \left( \frac{485}{108} \gamma B^4 + \frac{5}{1296} \gamma^3 B^{12} \right) \mu^4$$

$$(\theta_1 + \epsilon_0 \frac{M_w}{I_1 \Omega^2}), K_{26} = -\frac{1}{180} \gamma B^8 \mu^2 + \frac{1}{144 + \gamma^2 B^8} \left( \frac{18}{5} \gamma B^8 + \frac{13}{3240} \gamma^3 B^{13} \right) \mu^4$$

$$\left( \epsilon_1 \frac{M_w}{I_1 \Omega^2} \right), K_{27} = -\frac{1}{120} B^4 \mu + \left( \frac{1}{90} B^2 - \frac{1}{144 + \gamma^2 B^8} \frac{17}{1080} \gamma^2 B^{10} \right) \mu^3$$

$$\left( \eta_1 \frac{M_w}{I_1 \Omega^2} \right), K_{28} = \left( -\frac{1}{144} \gamma B^8 + \frac{1}{144 + \gamma^2 B^8} \frac{17}{90} \gamma B^8 \right) \mu^3$$

$$\left( \epsilon_2 \frac{M_w}{I_1 \Omega^2} \right), K_{29} = \frac{1}{144 + \gamma^2 B^8} \frac{34}{15} \gamma B^7 \mu^2$$

$$\left( \eta_2 \frac{M_w}{I_1 \Omega^2} \right), K_{30} = \left( -\frac{1}{12} B^3 + \frac{1}{144 + \gamma^2 B^8} \frac{17}{90} \gamma^2 B^{11} \right) \mu^2$$

$$\left( \left[ \frac{M_w}{I_1 \Omega^2} \right]^2 \right), K_{31} = \frac{1}{36} B^2 \mu^2 + \frac{7}{144} \mu^4$$

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TABLE I.—COEFFICIENTS  $K_i$  TO  $K_9$  OF ACCELERATING-TORQUE EXPRESSION FOR DIFFERENT TIP-SPEED RATIOS

Coefficient	Tip-speed ratio, $\mu$									
	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
$K_1$	0.502	0.526	0.558	0.598	0.646	0.702	0.766	0.840	0.923	1.017
$K_2$	.369	.421	.490	.579	.689	.824	.986	1.179	1.407	1.675
$K_3$	.268	.306	.356	.420	.500	.598	.714	.854	1.018	1.210
$K_4$	.022	.040	.066	.100	.145	.203	.276	.368	.481	.618
$K_5$	.062	.068	.096	.146	.212	.296	.403	.536	.700	.900
$K_6$	.012	.021	.035	.053	.077	.108	.147	.195	.255	.328
$K_7$	.112	.136	.206	.294	.330	.407	.496	.598	.716	.850
$K_8$	.042	.063	.090	.125	.169	.223	.290	.369	.465	.576
$K_9$	.031	.046	.066	.091	.123	.163	.212	.270	.339	.421